RECONSTRUCTION OF A PISTON TRANSDUCER BEAM USING MULTI-GAUSSIAN BEAMS (MGB) AND ITS APPLICATIONS

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INTRODUCTION

The modeling of ultrasonic wave propagation has become very important in the field of nondestructive inspection. Any ultrasonic simulation requires computation of the ultrasonic field produced by a transducer, and such computational models have become an important part of any ultrasonic simulator. Furthermore, these simulators require increasingly faster models for computation of the ultrasonic field for a given probe.

In recent years, many researchers have developed efficient models for ultrasonic wave propagation. Thompson and Lopes [1] developed a solution for the propagation of a Gaussian beam using the paraxial approximation. This solution had the advantage of being an analytical solution, and was very computationally efficient. Also, the solution could be used for focused probes. Minachi and Thompson [2] further developed an analytical solution for the propagation of Gaussian beams through curved interfaces in non-symmetrical planes. Both solutions have simple analytical forms that could be used to derive other NDE related solutions. Due to their computational efficiency, these solutions are ideal for practical NDE problems.

However, approximating a piston transducer as a Gaussian transducer may not be accurate in some cases. Therefore, a simple solution for a piston shaped beam is also desirable. Wen and Breazeale [3] reconstructed a circular piston shaped beam by adding several coaxial complex Gaussian beams. In this study, we used the same technique to describe the propagation of a piston shaped beam. This paper presents some applications of this technique in obtaining computationally efficient solutions to more complex problems.

BACKGROUND

Basic Theory

A piston shaped beam can be reconstructed by superposition of several coaxial complex Gaussian beams. Wen et. al. [3] used ten complex Gaussian terms to reconstruct a piston shaped beam. In a 2D space, where z is along the propagation direction and x is the transversed direction, this reconstruction of a piston shaped at $z = 0$ can be written as:

$$G(x)=\sum_{i=1}^{10} A_i\, e^{-B_i x^2} \tag{1}$$

where A_i and B_i are complex constants. Figure 1 shows graph of $G(x)$ along x direction. In Figure 1, the values of $G(x)$ were computed using Wen's constants A_i and B_i [3].

An analytical solution for the propagation of a single Gaussian beam can be found using the paraxial (or Fresnel) approximation. Thompson et. al. [1] derived such a solution in terms of:

$$\phi(x, y, z) = f(\phi_0, w_0, R_0) \tag{2}$$

where ϕ_0, w_0 and R_0 are a constant amplitude, half-width of the Gaussian beam, and the radius of the curvature of the wavefronts at z=0 respectively. If equation (2) is the solution to the wave equation within the paraxial approximation, then the linear sum of Gaussian beams would also be a solution. Therefore, one could write the solution as sum of many Gaussian beams.

$$\phi(x, y, z) = \sum_{i=1}^{10} \phi_i(\phi_{0i}, w_{0i}, R_{0i}) \tag{3}$$

where

$$\phi_i = A_i \tag{4}$$

$$w_{0i} = f_1(B_i) \tag{5}$$

$$R_{0i} = f_2(B_i) \tag{6}$$

By using equation (2) and equations (3-6) and Wen's constants A_i and B_i, the propagation of a piston shaped transducer is computed. To validate the results of such Multi-Gaussian beams, this technique was compared to the results of the Gauss-Hermite (GH) beam model. The Gauss-Hermite beam model is an approximate model which has been validated experimentally. Figure 2 shows the comparison of the Multi-Gaussian beam solution to the Gauss-Hermite solution.

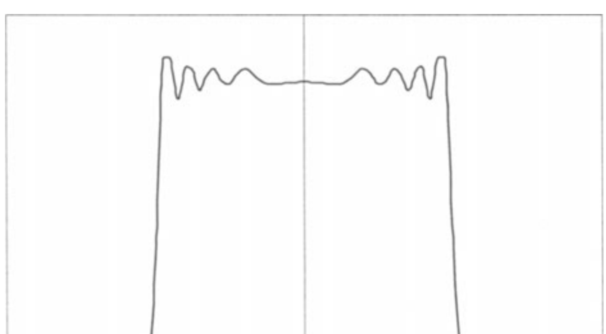

Figure 1. Beam profile of multi-Gaussian beam at initial plane (z = 0).

Propagation Through Curved Interfaces in Non-Symmetrical Planes

The solution given in equation (2) can be used to propagate the Gaussian beam through curved interfaces at oblique incidence. Thompson and Lopes [1] derived simple transformations for changes in beam width and radii of curvature of the phase fronts when a Gaussian beam reflects and transmits through an interface. However, they only considered incident angles in planes of symmetry of the interface. Minachi and Thompson [2] later analyzed the transmission and reflection of a Gaussian beam through curved interfaces for angles of incidence out of the planes of symmetry, also treating the subsequent propagation of the distorted beam.

The propagation in non-symmetrical planes has application to the inspection of BWR nozzles. In such inspections, ultrasound needs to be injected through more general incident angles. In these cases, representing a piston shaped beam as a Gaussian beam may not provide a very accurate solution. Therefore, there has been a need to propagate a piston shaped beam through curved interfaces in non-symmetrical planes. By applying the multi-Gaussian beam technique, one could define the piston shaped beam as a sum of several Gaussian beams, use the Minachi and Thompson solution [2], and propagate a piston shaped beam through curved interfaces in non-symmetrical planes.

To define the direction of the incident beam, Minachi et. al. defined a rectangular coordinate system with its origin at the point where the central ray of the incident beam intersect the interface. Then, the z-axis would be normal to the interface at the intersection point, and the z-x and z-y planes would contain the two principle radii of curvature of the interface. They defined the incident angle, θ, as the angle between the central ray and the z-axis. The skew angle, ϕ, was defined as the angle between x-axis and the projection of the central ray on x-y plane. Figure 3 shows the comparison of a Gaussian beam and piston shaped beam after propagation through a cylindrical interface at different skew angles.

Computation of Diffraction Correction

The diffraction correction, denoted here by D, describes the change in the response of a through-transmitted or pulse/echo signal due to beam spreading and focusing effects alone. The defining geometries for the single-medium case are illustrated in Figure 4a. We imagine having an ideal, isotropic, non-attenuating medium connecting the transmit-

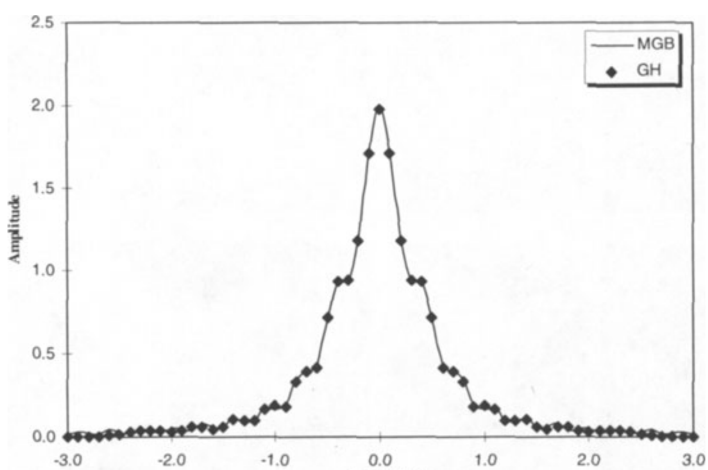

Figure 2. Comparison between multi-Gaussian beam (MGB) solution and Gauss-Hermite (GH) solution for a 2 MHz, 0.5 inch planar transducer propagating in water, z=6 cm.

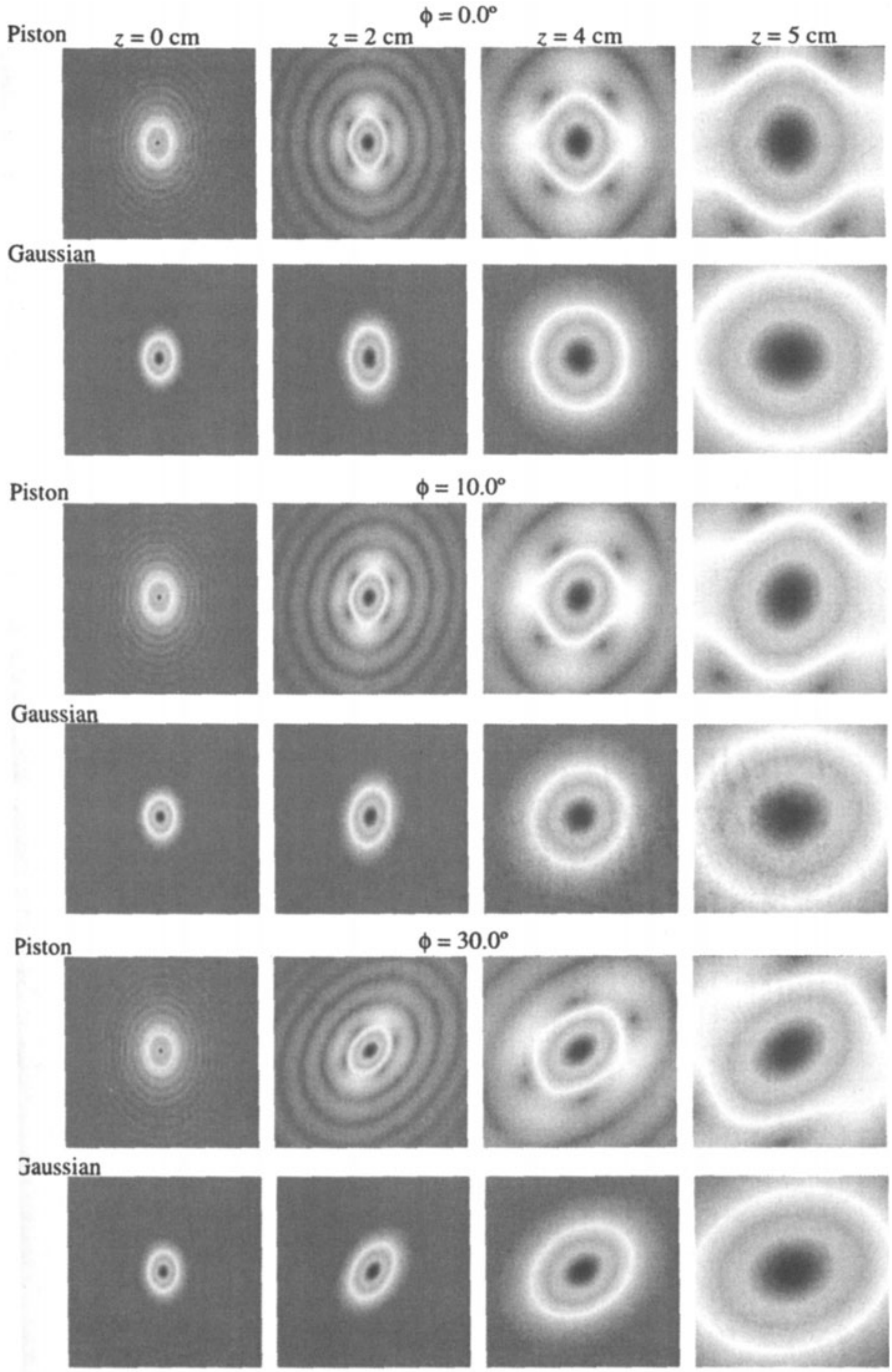

Figure 3. Comparison of beam profiles between a Gaussian and a piston transducer at different skew angles, ϕ. The beam is originated from a transducer (planar, 0.5" diameter, 2 MHz) in water (water path=5 cm) transmitted into an aluminum cylinder (OD=10 cm). The y-axis is along the cylinder's axial direction, and the incident angle, θ, is 10°.

ting and receiving transducers (which will be one and the same for the pulse/echo geometry). As the travel path is changed, the ultrasonic field impinging on the receiver will change as well, causing the output response to be modified. D describes this modification, and is usually normalized such that D = 1 when the separation between the transmitter and receiver is zero. In a straightforward way, the definition of D can be expanded to encompass multi-layered geometries such as that shown in Figure 4b.

The need to calculate D arises in various circumstances. For example, in attenuation measurements the attenuation coefficient is generally deduced by comparing two or more ultrasonic signals having different travel paths through the medium under study. For example, one might compare a front-wall echo with a back-wall echo, or compare two different back-wall reverberations. Only a portion of the measured difference between the two echoes will be due to the effects of attenuation. Interface transmission and reflection losses and diffraction losses will also be present and must be accounted for if the material attenuation is to be accurately estimated.

Another circumstance where the diffraction correction enters is in the application of inspection simulation models such as that of Thompson and Gray [4]. The objective may be to predict the absolute pulse/echo response from a small embedded defect in a component. One input to the model is often a "calibration" or "reference" echo reflected from a planar surface. That echo carries information about the strength, duration, and frequency content of the incident pulse, and it is analyzed to deduce the transducer's efficiency factor for the conversion of electrical energy to sound. When the reference echo is analyzed, the diffraction correction for the reference geometry must be calculated.

Formulas for the evaluation of diffraction corrections can be developed by integrating the arriving displacement field over the face of the receiver. Alternatively, Auld's reciprocity relationship can be used to obtain alternative expressions [5] in which one integrates the product of two fields (one from the transmitter and one from the receiver acting as a transmitter) over an intermediate surface. If the intermediate surface is chosen as the mid-plane between receiver and transmitter, the two fields are identical (except for propagation direction) and the calculation is simplified. For diffraction calculations associated with reflection at normal incidence, it is usually accurate to adopt two simplifying "paraxial" approximations: (1) to assume that a single component of the displacement field dominates (say the z component, U_Z) and that the two orthogonal components can be neglected; and (2) to assume that when derivatives of the displacement or stress fields are calculated the phase propagation term $\exp(-ikz)$ dominates, and hence that $\partial U_Z / \partial z \approx -ik\, U_Z$. When these two approximations are made, the evaluation of the diffraction correction reduces to the evaluation of the integral of the square of the incident velocity potential over the symmetric mid-plane. For the MGB model, the

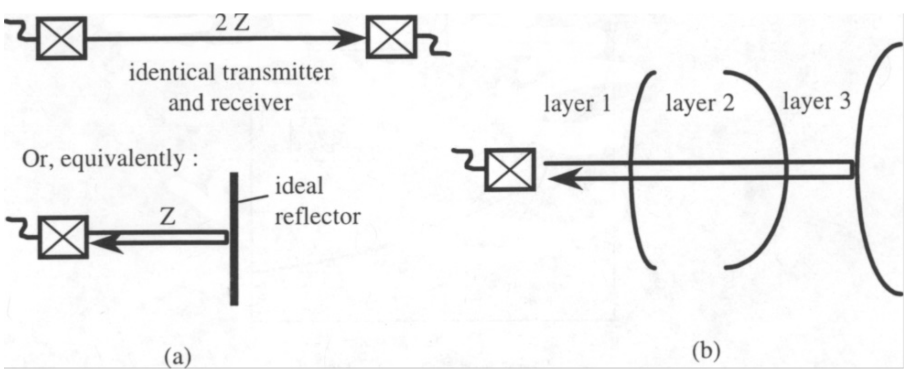

Figure 4. Geometries for diffraction correction calculations that can be treated using the multi-Gaussian beam model. (a) two equivalent simple geometries; (b) a multi-layered geometry with curved interfaces.

velocity potential generally contains N additive terms, and its square consequently contains N(N+1)/2 independent terms. Each of these terms is a complex Gaussian function whose surface integral over the mid-plane can be evaluated analytically. Thus the evaluation of the diffraction correction reduces to the summation of N(N+1)/2 relatively simple analytic terms. If the usual Gaussian-beam paraxial approximations are made for transmission through and reflection from curved interfaces [1], the MGB model can be used to obtain analytic formulas for diffraction corrections for situations like that shown in Figure 4b, in which there are multiple layers with curved interfaces. The formulas apply for either planar or bi-cylindrically (i.e., bi-spherically) focused transducers, and they can be used as long as the piezoelectric element has an elliptical shape. In all cases, the MGB expression for D is a summation of N(N+1)/2 analytic terms.

To demonstrate the accuracy of diffraction corrections calculated using the MGB model, we now present results for three test cases with different geometries. Here the MGM calculations use the ten-Gaussian expansion of Wen and Breazeale [3] and the aforementioned paraxial approximations. The first case, illustrated in the upper portion of Figure 5, considers the reflection from a planar interface of the beam from a circular, planar, piston transducer in water. The transducer diameter is assumed to be 0.25" and the oscillation frequency is 5 MHz. The diffraction correction as a function of the one-way water path has been calculated in two ways: (1) using the MGM model; and (2) using the analytic formula of Rogers and Van Buren for the Lommel's diffraction correction [6]. The latter, which is exact under the Fresnel approximation, is given by:

$$D = 1 - \exp\left(\frac{-2\pi i}{s}\right)\left[J_0\left(\frac{2\pi}{s}\right) + i J_1\left(\frac{2\pi}{s}\right)\right]; \quad s = \frac{4\pi z}{k a^2} \tag{7}$$

where a is the transducer radius, z is the one-way travel path, and k is the wave number in the diffracting medium. The magnitudes and real and imaginary parts of D are compared in Figure 5 for the two calculations. The agreement is seen to be excellent with the minor differences generally no larger than the line widths of the curves in the figure. In the second test case (Figure 6) the same transducer has been fitted with a spherical lens having a geometrical focal length denoted by F. As the distance to the reflecting plane (z) is varied, F is continuously adjusted such that $F = z$. For this geometry there is also an analytic expression for the Lommel diffraction correction, namely the negative of the

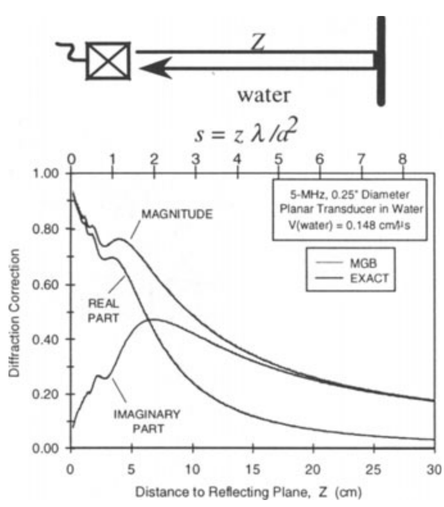

Figure 5. Diffraction correction for a circular, planar piston probe in water. The "exact" result uses the formula of Rogers and Van Buren.

complex conjugate of Eq. (12). Again we find excellent agreement between the MGB prediction for the diffraction correction and the "exact" result.

Our third test case is one which arises in the modeling of inspections of cylindrical titanium billets. Such inspections are often done using several transducers, each having a bi-cylindrically curved lens designed to produce an approximately circular focal spot at a specified depth within the billet, with the focal depths staggered so that each point in the billet is within the focal zone of at least one transducer. In this case we have assumed a 5-MHz, 2.35"-diameter, circular-element transducer with geometrical focal lengths of 8.1" and 25.7" in the *xz* and *yz* planes of Figure 7, respectively. The inspection waterpath is 3", resulting in a focal maximum just beyond the center of a 10"-diameter Ti 6-4 billet. One choice of reference signal is the back wall echo from a calibration specimen having the same entry surface curvature as the billet but having a flat back wall. In Figure 7 we have calculated the diffraction correction for such an echo as a function of the one-way metal travel path. In this case there are no other analytic formulas available. To test the MGB diffraction correction, we compare it with the result of using the Gauss-Hermite beam model to calculate the incident displacement field in the metal, and then numerically integrating the square of that field over the reflecting surface. Excellent agreement is seen.

In summary, we have found that the MGB model can be used to make accurate and rapid calculations of diffraction corrections for both simple and complex inspection geometries. It should also be possible to use the MGB model to obtain analytic expressions for the functions which describe the depth dependence of backscattered grain noise in metals. Those functions involve the integral of the fourth power of the absolute magnitude of the incident displacement field over a plane normal to the beam [7]. Because of the absolute value and fourth power operations, there will be many independent terms in the analytic expression that results from applying the MGB formalism. However, the computation time required for the evaluation of that lengthy analytic expression may be considerably shorter than that required for the numerical integration of the GH model field that is currently in use.

CONCLUSION

In this study, the propagation of a piston shaped beam using Multi-Gaussian Beam was developed. This technique was used to propagate a piston shaped beam through a

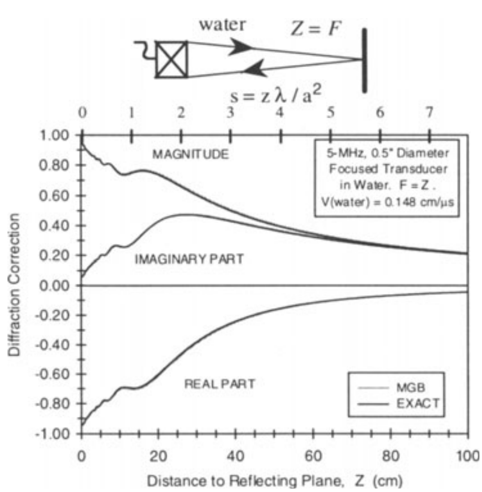

Figure 6. Diffraction correction for a circular, spherically focused piston probe in water. The "exact" result uses the formula of Rogers and Van Buren.

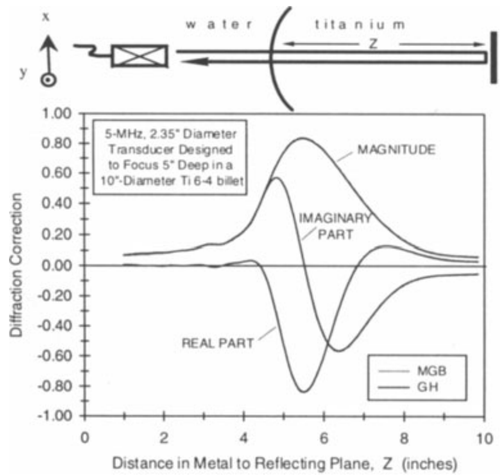

Figure 7. Diffraction correction arising in the inspection of a cylindrical titanium alloy billet.

curved interface (a cylinder) in non-symmetrical planes. The results showed a more descriptive beam profile of a piston transducer with side lobes.

Furthermore, MGB was used to compute the Diffraction Correction Factor. The results obtained by using MGB had excellent agreement with exact solution, but it could be used in more general problems where the exact solution does not apply. Also, MGB has the advantage of being computationally efficient compared to other techniques used for computation of Diffraction Correction Factor.

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